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## Research Report

# Strategic differences in algebraic problem solving: Neuroanatomical correlates

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### ABSTRACT

In this study, we built on previous neuroimaging studies of mathematical cognition and examined whether the same cognitive processes are engaged by two strategies used in algebraic problem solving. We focused on symbolic algebra, which uses alphanumeric equations to represent problems, and the model method, which uses pictorial representation. Eighteen adults, matched on academic proficiency and competency in the two methods, transformed algebraic word problems into equations or models, and validated presented solutions. Both strategies were associated with activation of areas linked to working memory and quantitative processing. These included the left frontal gyri, and bilateral activation of the intraparietal sulci. Contrasting the two strategies, the symbolic method activated the posterior superior parietal lobules and the precuneus. These findings suggest that the two strategies are effected using similar processes but impose different attentional demands.

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## 1. Introduction

Mathematical cognition has been studied in a number of neuroimaging investigations. Although most studies have focused on the representation of numbers and on arithmetic computation (e.g., Dehaene et al., 2003; Menon et al., 2002; Rivera et al., 2005), there has been some recent attempts to study more complex mathematical operations. Anderson et al. (2003), for example, found algebraic transformation to be subserved by the left posterior parietal region and the left dorsal lateral prefrontal cortex. Sohn et al. (2004) found differences in prefrontal versus parietal engagement, depend-

ing on whether algebraic questions were presented in a verbal or symbolic format. In this study, we examined another aspect of algebraic problem solving: differences resulting from the use of different problem solving strategies.

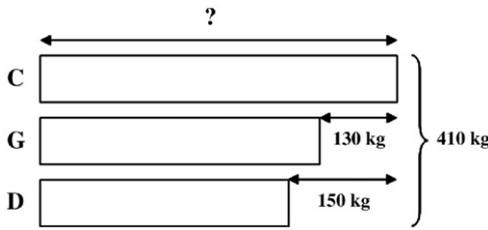
In school, algebraic problems are often presented as stories or word problems (see Fig. 1 for an example). Like students elsewhere, students in Singapore often find these questions difficult. To give them better access, primary schoolers (10–12-year olds) are taught a diagrammatic or model method. Students are taught to draw diagrams, normally made up of rectangles, to represent relationships presented in word problems (see Fig. 1). The rectangles represent unknowns.

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Abbreviations: SE, Symbolic experimental; SC, Symbolic control; HIPS, Horizontal segment of the intra-parietal sulcus; ME, Model experimental; MC, Model control; PSPL, Posterior superior parietal lobules



**Fig. 1 – A model solution to the question: A cow (C) weighs 150 kg more than a dog (D). A goat (G) weighs 130 kg less than the cow. Altogether the three animals weigh 410 kg. What is the weight of the cow?**

Students are expected to solve for the unknowns by analysing the quantitative relationships between the rectangles.

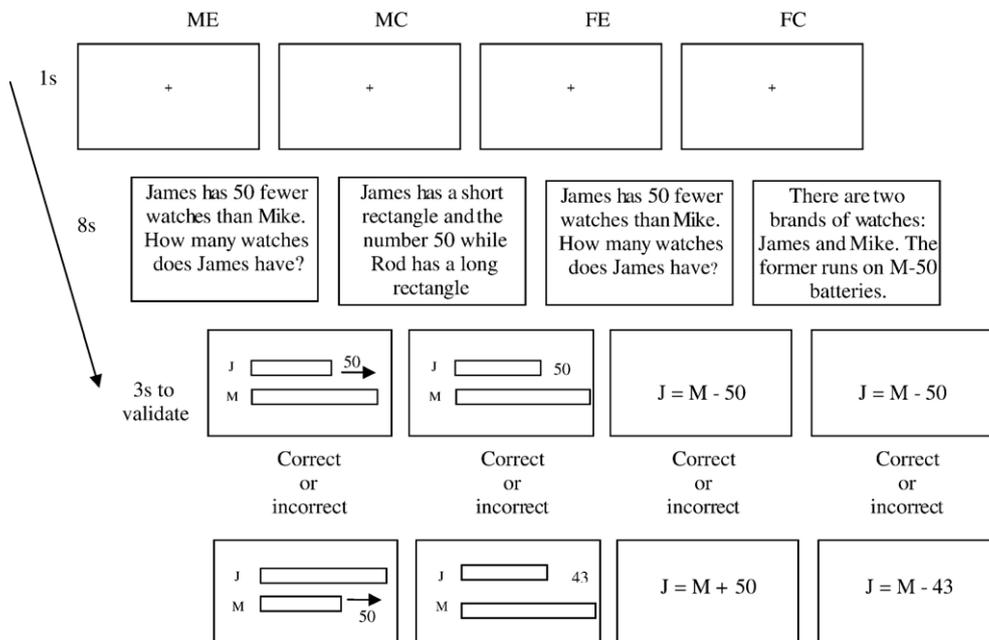
As students’ success with word problems are affected by whether they understand the questions and whether they can transform the questions into equations or models (Carpenter et al., 1988; Kintsch and Greeno, 1985; Mayer, 1992; Riley and Greeno, 1988; Stacey and MacGregor, 1999; Verschaffel and De Corte, 1993), a strategy that requires explicit consideration of relevant relationships should promote accuracy. Indeed, Lewis (1989) showed that even college students benefited from training involving the use of pictorial representations.

The present study is part of an effort to examine whether the model method assists in the acquisition of formal or symbolic algebra (which, in Singapore, is taught in secondary or high school). Previous studies suggest teaching the model method may have both positive and negative effects. Findings from Khng and Lee (submitted) showed that even when instructed to use only symbolic algebra, students from secondary schools often exhibited intrusion errors and used the model method. Although such behaviour could be seen as adaptive in that students were using an alternative heuristic that was more accessible, many teachers saw the same behaviour in a negative

light. In interviews and in feedback from in-service training, many secondary school teachers viewed the model method as childish, non-algebraic, and thought it a hindrance to the teaching of symbolic algebra (Ng et al., 2006).

A full answer to whether the model method assists in the acquisition of symbolic algebra will need to address cognitive, motivational, and pedagogical issues. In this study, we focused on the cognitive issues. We examined whether the model method and symbolic algebra were subserved by similar processes in adults with similar behavioural competency across the two methods. In terms of surface characteristics, the two methods seem to engage different types of information. The model method makes use of pictorial and alphanumeric information in depicting information. Symbolic algebra makes use of alphanumeric information only. Despite such differences, skilled mathematicians consider the two methods to be equivalent. The main difference being the way in which unknowns are represented: as rectangular boxes in the model method and as letters,  $x$  or  $y$ , in symbolic algebra. Because the model method has been part of the national curriculum in Singapore for over a decade, traditional programme evaluation techniques are of little assistance. In this study, we used functional magnetic resonance imaging (fMRI) to examine similarities and differences in processes that subserved the two strategies.

Information processing models of word problem solving guided the construction of experimental tasks. Most descriptions stipulated two stages: problem representation and problem solution (Bobrow, 1968; Briars and Larkin, 1984; Lewis, 1989; Riley and Greeno, 1988; cf. Koedinger and MacLaren, 2002). In a recent rendition, Mayer and Hegarty (1996) expanded these stages further. They argued that information such as quantitative relationships between protagonists is first extracted from the word problem. Pre-existing knowledge relevant to the problem is then activated and is integrated with the extracted information. Procedure



**Fig. 2 – Experimental procedure.**

**Table 1 – Mean accuracy (%) and reaction times (ms) for all conditions**

Conditions	Mean accuracy (S.D.)	Mean RT (S.D.)
Model experimental	94.52 (6.33)	1223.46 (191.80)
Symbolic experimental	96.06 (4.32)	1175.19 (207.09)
Model control	99.07 (1.35)	1152.52 (194.09)
Symbolic control	94.29 (4.83)	1132.27 (144.90)

needed to compute the solution is then planned. The final step involves computation of values. In this study, we focused on problem representation. Participants were asked to transform information from text to structure using either the model method or symbolic algebra. This was followed by a validation task in which participants were asked to compare the presented solution with the one they had in mind (see Fig. 2).

From a functional neuroanatomical perspective, because quantitative relationships were depicted and needed to be considered regardless of problem solving strategy, both methods were expected to activate areas associated with quantitative processing. In a recent paper, Dehaene and his colleagues (2003) proposed a triple-code model in which the horizontal segment of the intra-parietal sulci (HIPS) was involved in cross-modal quantitative processing. Other studies have found the HIPS activated in magnitude comparison tasks involving different stimuli (lines, angles, versus Arabic numbers, Fias et al., 2003), in numerosity habituation tasks involving dot arrays (Piazza et al., 2004), and in mental arithmetic tasks involving Arabic numbers versus array of dots (Venkatraman et al., 2005).

Findings from Terao et al. (2004) are of direct relevance to this study. Terao and his colleagues presented participants with three-protagonist algebraic problems. A generator or base variable was first defined, e.g.,  $x = A$ . This was followed by specification of quantitative relationships between the generator and other protagonists in the problem. Participants were asked to depict the relationships between the various protagonists using either a mental number line or an algebraic equation. In comparison to the generator assignment stage, i.e.,  $x = A$ , the mental number line condition was associated with bilateral activation in the HIPS. The HIPS was also activated in the equation condition but was largely left lateralised. This finding was surprising as previous studies had found the HIPS insensitive to modality based differences (e.g., Fias et al., 2003). However, it should be noted that Terao et al.'s finding was based on a preliminary analysis and no direct comparison was made between the two conditions.

Another issue of interest was whether the two methods differed in terms of working memory or attentional requirement. The model method is taught in primary school because it is believed to give students easier access to challenging word problems. Whether this is effected by the more concrete nature of models (Kho, 1987), reduced attentional requirement, or some other means is unknown. If symbolic algebra does indeed have higher working memory or attentional demands, it can be expected to result in greater activation in frontal and parietal areas previously found to subserve these processes (e.g., Owen et al., 2005).

According to Kieran (2004), processes central to algebraic problem solving include analyzing the quantitative relationships between protagonists and modelling the structure of such

relationships. By asking participants to transform text based questions into either equations or models, this study was designed to identify differences in such processes across the two strategies. Because the two experimental conditions also differed in solution formats, the data were potentially confounded by format specific differences. To assist in interpretation, we included two format-specific control conditions. Participants were presented with explicit verbal descriptions of either (a) the rectangles used in constructing models or (b) the alphanumeric

**Table 2 – Talairach coordinates of activation maxima for the model and symbolic methods: (SE > SC) and (ME > MC)**

Brain regions	Talairach coordinates							
	Left hemisphere				Right hemisphere			
	x	y	z	t	x	y	z	t
Medial superior frontal gyrus (BA8) <sup>a,b</sup>	-6	14	49	6.65				
Middle frontal gyrus (BA9) <sup>a,b</sup>	-42	11	34	5.60	30	2	55	4.30
Middle frontal gyrus (BA9) <sup>a</sup>	-21	-7	43	4.40				
Middle frontal gyrus (BA9) <sup>b</sup>					45	11	34	8.40
Frontal pole (BA10) <sup>a,b</sup>	-39	41	12	7.60				
Horizontal intraparietal sulcus (BA40) <sup>a,b</sup>	-48	-45	40	5.00	36	-49	34	4.45
Precuneus (BA7) <sup>a, b</sup>	-9	-67	38	5.55				
Recuneus (BA7) <sup>b</sup>					9	-62	43	4.25
Angular gyrus (BA39) <sup>a,b</sup>					30	-57	34	4.75
Cingulate gyrus (BA32) <sup>a</sup>					18	11	40	5.10
Cingulate gyrus (BA32) <sup>b</sup>					12	17	34	4.85
Superior temporal gyrus (BA39) <sup>b</sup>	-48	-49	7	5.30				
Middle temporal gyrus (BA21) <sup>b</sup>	-60	-37	1	4.40				
Middle temporal gyrus (BA20) <sup>b</sup>	-51	-40	-14	5.75				
Calcarine sulcus (BA17) <sup>a</sup>	-18	91	1	6.65				
Lingual gyrus (BA18) <sup>b</sup>	-4	-67	4	4.50				
Fusiform gyrus (BA37) <sup>b</sup>					42	-50	-17	6.90
Caudate <sup>a,b</sup>					11	2	10	4.55
Caudate <sup>b</sup>	-21	-7	22	5.10				
Anterior insula <sup>a,b</sup>	-24	20	7	5.70	27	23	5	6.00
Anterior thalamus <sup>a,b</sup>	-13	-4	4	4.25				
Lateral posterior thalamus <sup>b</sup>					15	-22	16	5.20
Ventral posterior thalamus <sup>b</sup>					18	-19	1	4.70

<sup>a</sup> = Activations for SE > SC.

<sup>b</sup> = Activations for ME > MC.

<sup>a,b</sup> = Areas activated in both (SE > SC) and (ME > MC). For these regions, coordinates reflect activation maxima found in the conjunction analysis.

characters used in symbolic algebra completion (see Fig. 2, model control, MC, and symbolic control, SC, for examples). They were asked to use these descriptions to construct appropriate mental representations. Descriptions used in the control conditions were designed to produce model or symbolic representations identical in form to those produced in the experimental conditions. Prior to scanning, participants were given detailed instructions and practice. Although numbers were included in the control descriptions, they were presented as nominal labels. Unlike the experimental conditions, no numeric consideration was required for task completion.

## 2. Results

### 2.1. Behavioural findings

Paired-sample t-tests conducted on data from the experimental conditions revealed no significant differences in either accuracy or reaction time between the two strategies. Data from the control conditions revealed a reliable difference in accuracy with more accurate responses in the model than in the symbolic condition (see Table 1 for means and standard deviations).

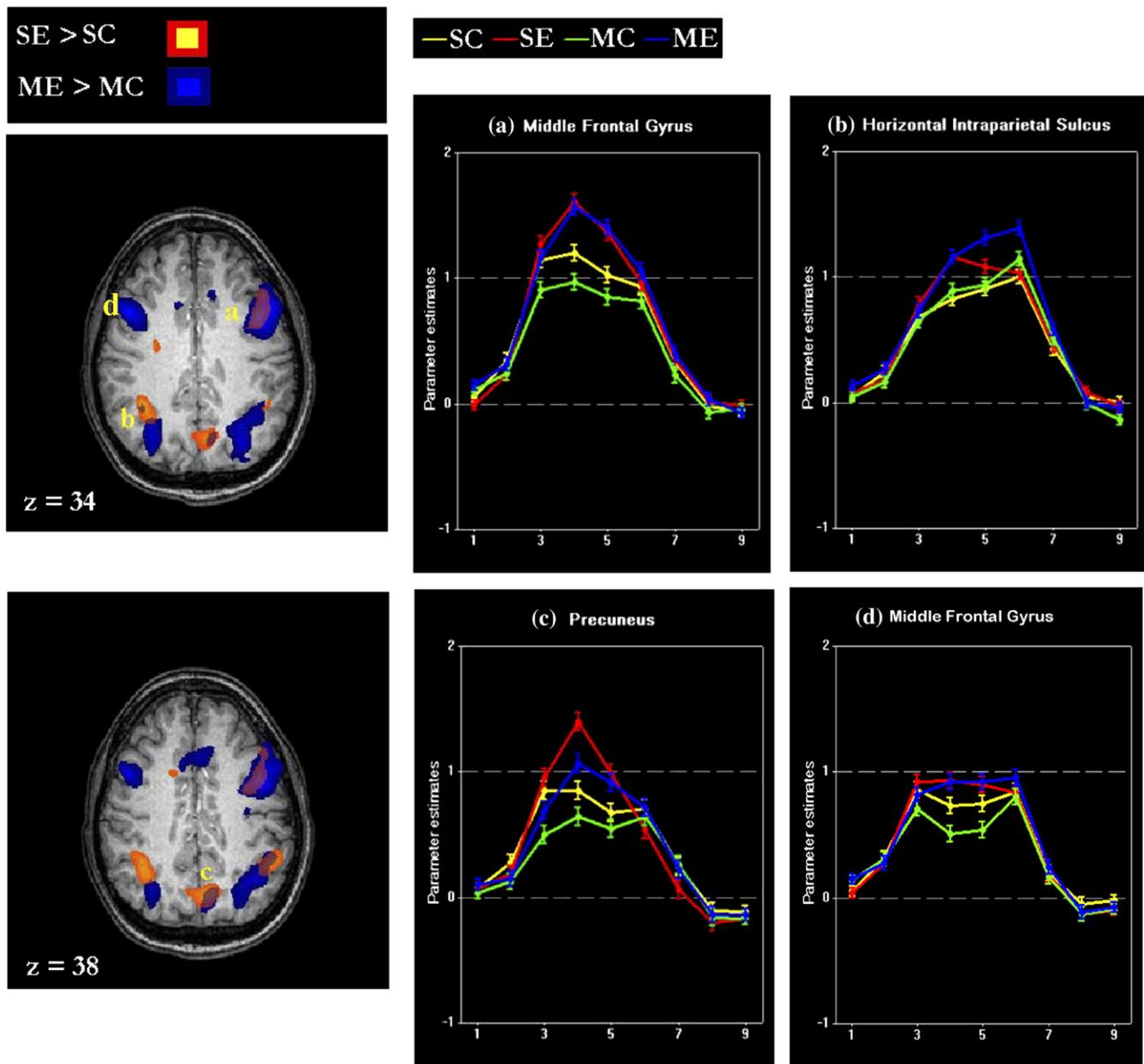


Fig. 3 – Activation map for (ME>MC) and (SE>SC). Overlapping areas, in a darker shade, reflect activations for the conjunction between the two contrasts. A threshold of  $p < 0.001$ , uncorrected, was used to determine whether a voxel was activated. The left side of each transverse slice represents the right side of the brain.

## 2.2. fMRI findings

### 2.2.1. Similarities between the model and symbolic methods

To find common areas activated for both the model and the symbolic methods, we conducted a conjunction analysis involving the model experimental (ME), model control (MC), symbolic experimental (SE), and symbolic control (SC) conditions. In this analysis, contrasts between the experimental and control conditions were first estimated for each method, i.e., (ME>MC), (SE>SC). These two contrasts enabled us to identify regions specifically activated by different problem solving strategies by excluding regions involved in processes unrelated to problem solving, such as reading the stimuli. They also exclude processes specific to the generation of pictorial or text images. We then identified areas that were activated by both solutions using a conjunction analysis, i.e., (ME>MC) and (SE>SC). Based on Nichols et al. (2005), areas were identified as commonly activated only if each contributing contrast exceeded the statistical threshold ( $p < 0.001$ , uncorrected). Reported *t* values corresponded to the contrast with the smaller signal difference.

The conjunction analysis revealed activation in the frontal cortex, both left lateralised and along the midline. It also revealed bilateral activation in the HIPS (see Table 2 and Fig. 3 for details). These findings suggest the experimental conditions engaged working memory and magnitude comparison processes more extensively than did the control conditions.

### 2.2.2. Differences between the model and symbolic methods

We were interested in whether different neural systems subserved the model versus symbolic methods. An initial analysis involving the symbolic and model experimental conditions – SE>ME – showed that the symbolic method was associated with activation in the precuneus and bilateral posterior superior parietal lobules (PSPL). This finding suggests the symbolic condition recruited attentional processes more extensively than did the model method. Also activated were various loci in the visual processing area and in the basal ganglia. The model condition did not activate any areas above or beyond those activated by the symbolic condition (see Table 3 and Fig. 4 for details).

To identify differences between the model and symbolic methods that are specific only to the methods and not to processes required for generating different solution formats, an interaction analysis of the sort (ME>MC)>(SE>SC) is necessary. These interactions, however, can arise from both differences in experimental as well as control tasks. Because behavioural differences were observed in performance accuracy between the two control tasks, results from such analyses are likely to be confounded by interactions involving differences in control task activation (see Fig. 3, time course d, for an example of activation differences resulting from the control but not the experimental conditions).

To avoid such confounds, we conducted a conjunction analysis involving three contrasts: (SE>ME) and (SE>SC) and (SE>MC). These analyses are more conservative and considered only areas in which activation in each experimental condition was greater than those in both control conditions. A subset of areas found in the initial analysis was found activated by the symbolic method (see Table 3 for details). They included the precuneus, left cuneus, and right caudate. A corresponding analysis was conducted to identify areas associated with the model method: (ME>SE) and (ME>SC) and (ME>MC). However, no area was activated only by the model method.

**Table 3 – Talairach coordinates of activation maxima in the symbolic method (SE>ME)**

Brain regions	Talairach coordinates							
	Left hemisphere				Right hemisphere			
	x	y	z	t	x	y	z	t
Precuneus (BA7) <sup>a</sup>	0	-64	43	5.55				
Posterior superior parietal lobule (BA7)	-24	-70	40	4.80	15	-64	37	5.40
Posterior cingulate (BA 31)	-12	-55	28	4.35				
Cuneus (BA30/17) <sup>a,b</sup>	-27	-73	13	5.40	14	-91	4	5.05
Lingual gyrus (BA 17/18)	-21	-88	4	4.75				
	-27	-70	1	5.25				
	-9	-91	14	7.55	7	-85	-8	5.85
Fusiform gyrus (BA 18/19/36)					24	85	-14	5.10
					27	-70	-17	4.65
					21	-44	-14	4.85
Middle occipital gyrus (BA 18)					27	-85	4	5.85
Inferior occipital gyrus (BA 18)	-36	-82	-17	5.10				
Superior temporal gyrus (BA 22)					54	-49	13	4.30
Inferior temporal gyrus (BA 20)	-51	-25	-19	5.00				
Caudate <sup>a</sup>					12	14	10	4.65
Globus pallidus					15	5	-5	4.90

<sup>a</sup> Areas activated for the conjunction: (SE>ME) and (SE>MC) and (SE>SC).

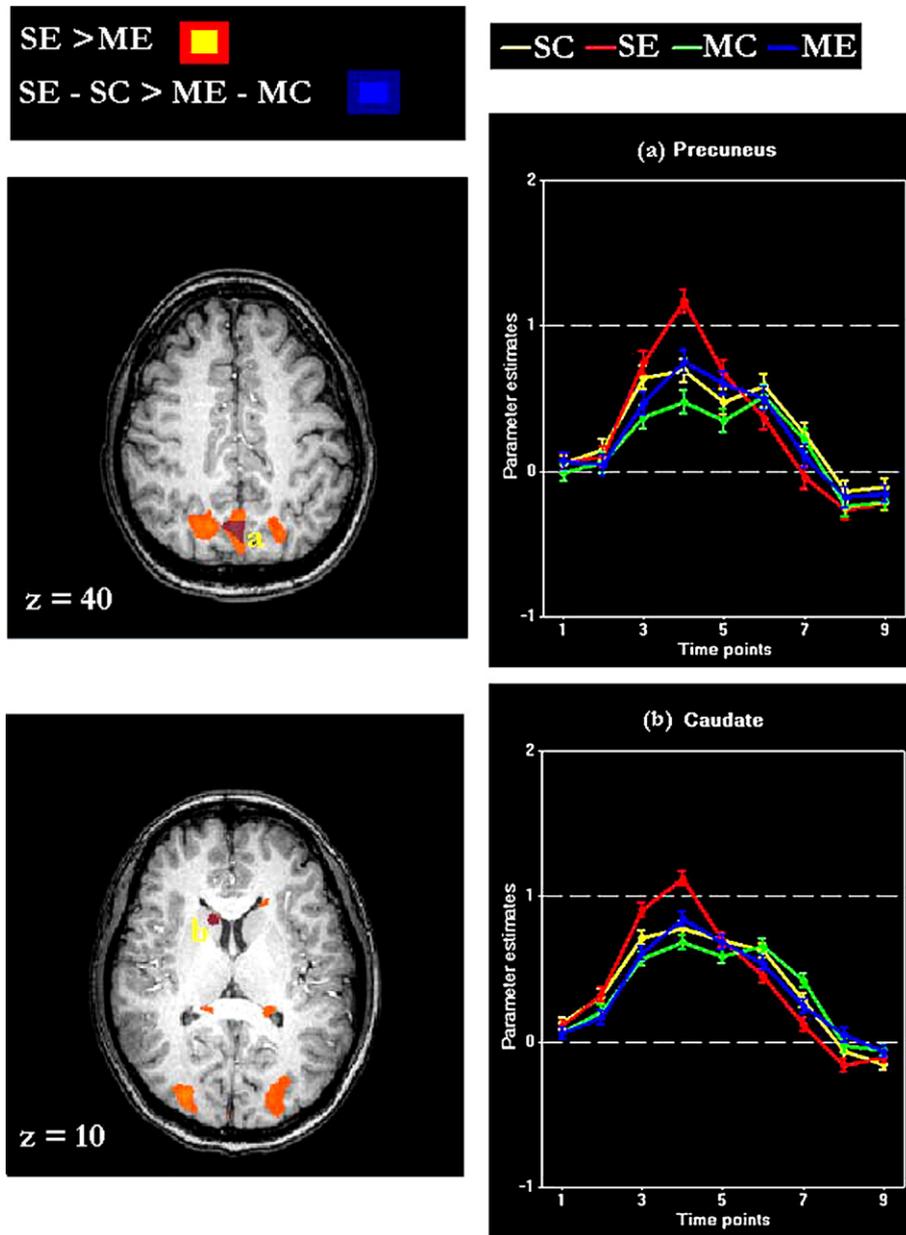
<sup>b</sup> Activation for the conjunction: (SE>ME) and (SE>MC) and (SE>SC) was only found in the left hemisphere.

## 3. Discussion

This study was motivated by queries regarding cognitive processes that subserved two methods for representing algebraic word problems. Both the symbolic and model methods require more extensive magnitude comparison and working memory engagement than do the control conditions. This is evident by findings from a conjunction analysis showing bilateral activation in the HIPS and left lateralised activation in the frontal gyri respectively. Although both methods draw on attentional resources, the symbolic method is more demanding. Activation in the precuneus and PSPL, both associated with attentional processes, were found to be stronger in the symbolic than in the model condition.

### 3.1. Similarities between the model and symbolic methods

As expected, the HIPS were activated bilaterally in both the model and symbolic conditions. This region has previously been associated with a mental number line (Dehaene et al., 2003) and is active in both symbolic and non-symbolic



**Fig. 4 – Activation map for (SE>ME) and the conjunction between (SE>SC) and (SE>MC). Overlapping areas, in a darker shade, reflect the conjunction between (SE>ME) and (SE>SC) and (SE>MC). A threshold of  $p < 0.001$ , uncorrected, was used to determine whether a voxel was activated. The left side of each transverse slice represents the right side of the brain.**

numeric processing tasks (Fias et al., 2003; Piazza et al., 2004). In the present study, HIPS activation is likely related to participants engaging in magnitude comparison to help verify which protagonist possessed more target objects.

It is possible HIPS activation merely reflects exposure to numbers. Eger et al. (2003) showed mere presentation of numbers activated HIPS. However, we deem this explanation less likely. Exposure to numbers was controlled both in question presentation and in the response verification phase of the procedure. The same questions were used in the two experimental conditions. Furthermore, HIPS activation was found in contrasts in which activation from the control conditions were subtracted from the experimental conditions. Because the same amount of numeric stimuli was presented

in the experimental and control conditions, the resulting difference in HIPS activation cannot be attributed to differential exposure to numeric stimuli.

Greater activation in the frontal gyri is indicative of greater working memory or executive involvement in the experimental than in the control conditions. Similar areas were identified in two working memory meta-analyses (Owen et al., 2005; Wager and Smith, 2003). The area labelled as dorsolateral prefrontal in Owen et al. overlaps with the middle frontal area found in the present study and was characterised by Owen et al. as being involved in reorganising material into pre-existing knowledge structures: a process that seems central to the transformation of information embedded in word problems to either models or equations. Wager and Smith (2003)

found evidence suggesting that this area was closely associated with continuous updating. Their study also found left lateralised activation is most frequently associated with verbal working memory tasks. This interpretation is consistent with findings from the behavioural literature showing that verbal working memory tasks predicted reliably individual differences in algebraic problem solving (Lee et al., 2004).

Also activated in the experimental conditions were the right precuneus and the anterior insula. Although the precuneus has most commonly been associated with selective attention, in Wager and Smith's (2003) meta-analysis, it was activated in all three studied executive functions: updating, remembrance of order information, and manipulation. These findings suggest the precuneus may subserve the attentional functions of executive functioning and are consistent with theories of working memory that emphasise close linkages between working memory and attention (e.g., Baddeley and Logie, 1999; Cowan, 1999). The insula is commonly associated with motivational and affective aspects of learning and memory. In this study, there are few reasons to suspect that the experimental condition engaged affective processing any more so than did the control conditions. Recent reviews on the functions of the insula suggest a broader role that includes verbal working memory and selective visual attention (Augustine, 1996; Bamiou et al., 2003).

### 3.2. Differences between the model and symbolic methods

Regarding differences, the symbolic condition was found to activate the caudate more so than did the model condition. According to the ACT-R model (Anderson et al., 2003), activation in the basal ganglia may reflect retrieval of procedural memory. In the context of this study, this finding suggests that construction of algebraic equations is more reliant on procedural retrieval. In addition to the caudate, the PSPL and an area of the precuneus to the left of that found in the similarity conjunction were activated in the symbolic algebraic condition. In addition to its role in imagery and episodic memory retrieval (Fletcher et al., 1995; Lundstrom et al., 2003), the precuneus has been found associated with other processes. Zago and Tzourio-Mazoyer (2002), for example, found the left precuneus activated in a complex arithmetic task as compared to a visual spatial working memory task. Behrmann et al. (2004) found goal directed non-spatial shifts of attention to be subserved by the precuneus, which was activated when participants shift their attention between two dimensions of an input. Dehaene and his colleagues (2003) classified both the PSPL and the precuneus as being part of a parietal number processing circuit that contributes to attentional selection and orientation. In the present study, activation of the precuneus suggests additional resources devoted to attention orientation or retrieval of relevant information are required in generating algebraic equations from word problems.

One criticism of this conclusion is that differences may be related to the solution format in the two conditions, i.e., verbal versus pictorial, rather than algebraic versus model. Although such format specific differences are intrinsic to the two strategies, we differentiated these differences versus those that are not related to solution format by using format specific controls. Findings from a simple subtraction involving the two experimental conditions showed that occipital areas were

activated in the algebraic condition. This suggests participants spent more time viewing the questions in the algebraic condition. One concern is that other parietal activation, particularly those in the PSPL and the precuneus, are artefacts of this difference. Findings from the SE>ME conjunction analysis, in which activation from the control conditions was subtracted, suggest activation in the PSPL may indeed be related to these activities. Activation in the precuneus, however, remains reliable.

### 3.3. Conclusions

Both the model and symbolic methods activated similar areas in the frontal gyri and HIPS. Differences were found in the precuneus and caudate regions. These findings suggest that one reason for the efficacy of the model method is its lower demand on attentional resources. These findings are particularly important because differences were found in spite of behavioural equivalence: all participants were screened for accuracy and had to fall within a narrow inclusion criterion. Although the efficacy of pictorial strategies has been demonstrated before (Lewis, 1989), this is the first study to show that their efficacy is not due to participants' competency in using the strategies.

Given the extensive overlap in activation across the two methods, it is tempting to conclude that they engaged similar cognitive processes. This should be reassuring for teachers who may be concerned that the model method is non-algebraic. For readers interested in possible pedagogical implications, we have some caveats. First, though the findings are not consistent with a view specifying extensive differences across the two methods, differences suggestive of differential engagement of similar processes were found. Second, the data were collected from adults with similar behavioural competency across the methods. Although we see this as a particular strength of the study, whether the same findings will hold for children or for participants with different competency across the two methods will require further studies. One way to examine this issue is to use a parametric design in which participants with graded levels of competency on the two methods are compared. Third, although the study was motivated by queries on whether the model method aided children in acquiring symbolic algebra, we were only able to address one aspect of this question. A full answer will require additional studies addressing associated pedagogical and motivational concerns. Even amongst the cognitive issues of concern, we focused only on problem representation. Whether differences exist in later stages of problem solving – in computing solutions from algebraic versus model representations – remain to be addressed.

## 4. Experimental procedures

### 4.1. Participants

The sample contained 18 right-handed volunteers (10 males, 20 to 25 years of age). Potential participants were screened to ensure they could achieve more than 90% accuracy on a task similar to those used in experiment trials. Furthermore, we selected only those participants who exhibited less than 5%

difference in accuracy between the two methods. This ensured that differences in activation were not due to variation in participants' competencies in the two methods. To further ensure that differences were due to intrinsic differences between the two methods and to minimise the impact of differences in speed or stage of acquisition, we used adults in this first study. All participants gave informed consent and were treated in accordance to applicable ethical guidelines.

#### 4.2. Design and procedure

The study was based on a 2 (Strategies: model vs. symbolic) × 2 (Condition: experimental vs. control) within subject design. In total, participants completed 144 trials, divided into 6 alternating symbolic or model blocks. Within each block, participants were presented with both experimental and control trials. In each trial, the algebraic problem, in text form, was presented for 8s. Instructions presented prior to each block asked participants to create and to hold in mind either a model or a symbolic representation of each problem. Depending on the block, either a model or a symbolic representation of the problem was presented at the end of the problem presentation period. Participants were given 3s and were asked to compare and validate the presented representation against the one they had in mind. Their decisions were expressed using a response button. The representation disappeared on key press and was replaced by a blank screen for the remainder of a 3s response period (see Fig. 2). Participants were introduced to the procedure and were given practice trials prior to entering the scanner.

Half of the presented representations were correct and the remainder incorrect. Participants were told there were 2 possible types of errors: interchanged relationship (e.g., specifying a greater-than rather than a less-than relationship between protagonists,  $J=M+50$  rather than  $J=M-50$ ) and number errors (specifying the wrong numeric magnitude, e.g.,  $J=M-43$  rather than  $J=M-50$ ). Two types of error were used to maintain alertness and to increase task difficulty. If the participants did not respond, the trial was scored as incorrect. Each participant went through a practice session before they entered the scanner. This session exposed participants to the procedure and to the types of problems they were going to encounter.

#### 4.3. Imaging protocol

Functional images were collected using a Siemens 3T Allegra system. Stimuli were projected onto a screen at the back of the magnet. Participants viewed the screen via an angled mirror fastened to the head coil. 36 axial slices approximately parallel to the SC-PC line were acquired using an interleaved gradient-echo echo planar imaging sequence (TR=3000 ms, TE=30 ms, pixel matrix=64×64; FOV=192 mm; 3 mm thickness, 0.3 mm gap). High resolution co-planar T2 anatomical images were acquired in the same orientation. High resolution anatomical reference images were acquired using a 3D-MPRAGE sequence.

#### 4.4. Data analysis

Functional images were pre-processed and analyzed using Brain Voyager QX version 1.26 (Brain Innovation, Maastricht, Holland). Gaussian smoothing kernel of 8mm FWHM was applied in the

spatial domain. A high-pass frequency filter was applied following linear trend removal. The functional images were aligned to the co-planar high resolution T2 images. The image stacks were then aligned to high resolution 3D images of the brain. The resulting data set was transformed into Talairach space.

Functional data were analysed at group level with a general linear model. Each trial was modelled using nine finite impulse response predictors, spanning a total of 27s from trial onset: 11s for the actual task and an additional 16s for the hemodynamic response to decay to baseline. A random effects analysis was used to identify significant differences across conditions. Region-of-interest (ROI) based analyses were performed on voxels that were identified as reliably activated. Data from the fourth to sixth predictor or time point were analysed and revealed similar patterns of findings. Unless otherwise indicated, all reported findings are based on the fourth time point. A statistical threshold of  $p < 0.001$  (uncorrected) and a cluster size of 27 contiguous voxels were used for the identification of activated clusters. Reported ROIs are maxima identified with a bounding cube of 10 mm surrounding the activation peak for each ROI.

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